

# Graph-Based Specification and Automated Construction of ILP Problems

GCM'22



Real-Time Systems Lab

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Dept. of Electrical Engineering and Information Technology

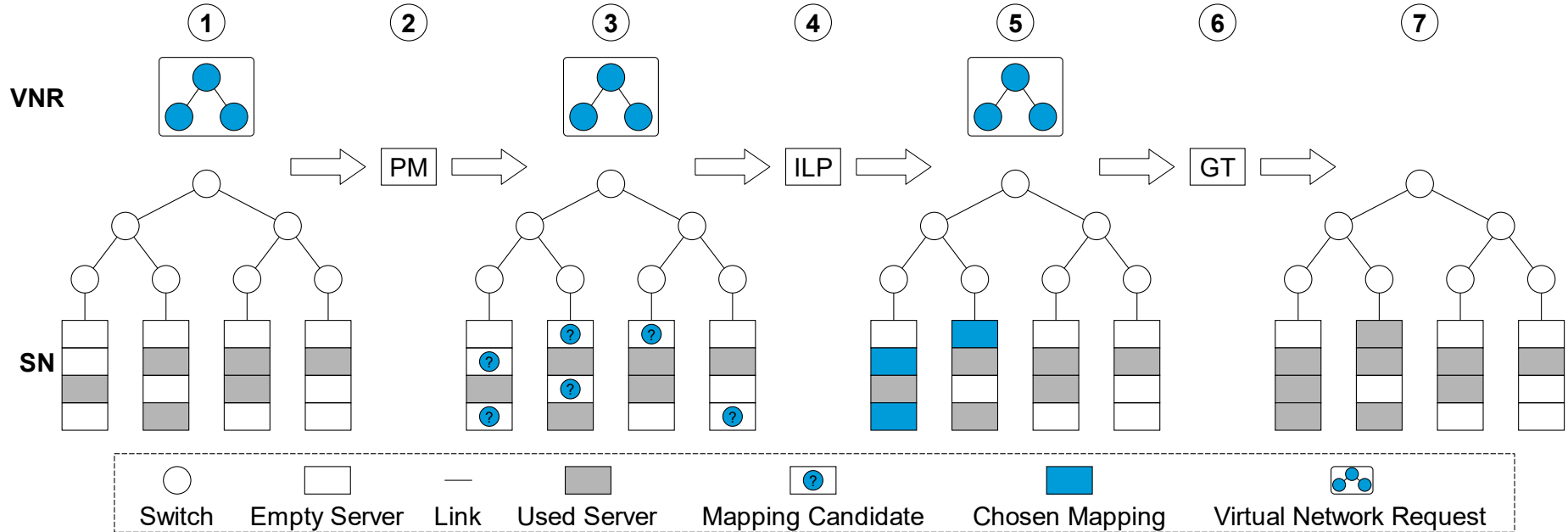
Dept. of Computer Science (adjunct Professor)

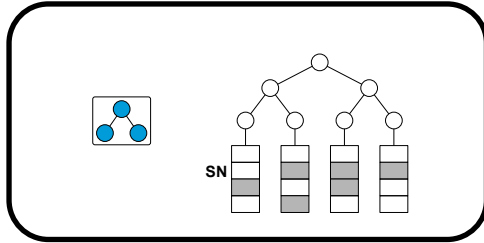
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**Maximilian Kratz**

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# Virtual Network Embedding (VNE)





## ➤ Graph Mapping Problem

- Input/Output: Graphs
- Constraints
- **Goal:** Optimize a given cost function (objective)
- For every new scenario:
  - Implement an ILP generator "by hand"
  - Connect the ILP generator with the GT framework
- Requires ILP expertise + Highly error-prone



How to solve such problems until now?

- Build yourself a specific tool for one problem domain
  - For example, with (M)ILP: iDyVE<sup>[1]</sup>
- Model synchronization tools
  - Triple Graph Grammars (e.g., with eMoflon-IBeX<sup>[2]</sup> or eMoflon-Neo<sup>[3]</sup>)
  - Janus Transformation Language (JTL)<sup>[4]</sup>
- Others (example): MOMoT<sup>[5]</sup>



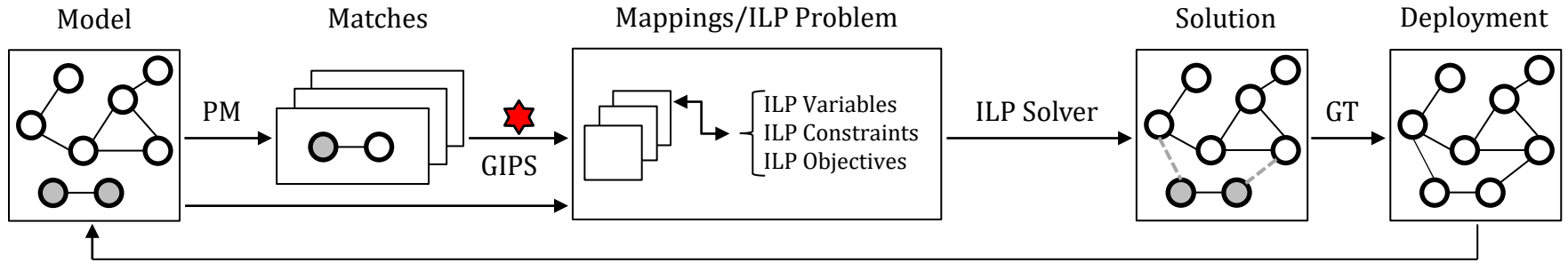
# Our Idea



- Goal: Convenient specifying and efficient solving of graph mapping problems
  - "Without" ILP expertise
  - With little implementation effort
- Domain-Specific Language (DSL) for specifications
  - Based on eMoflon-GT
  - Integrates ILP constraints, patterns and GT rules
  - Adds specification of objectives
- Framework generates ILP generators using a given specification
- Combines eMoflon::IBeX-GT, HiPE<sup>[6]</sup>, an ILP generator and an ILP solver
- A tool to build other tools



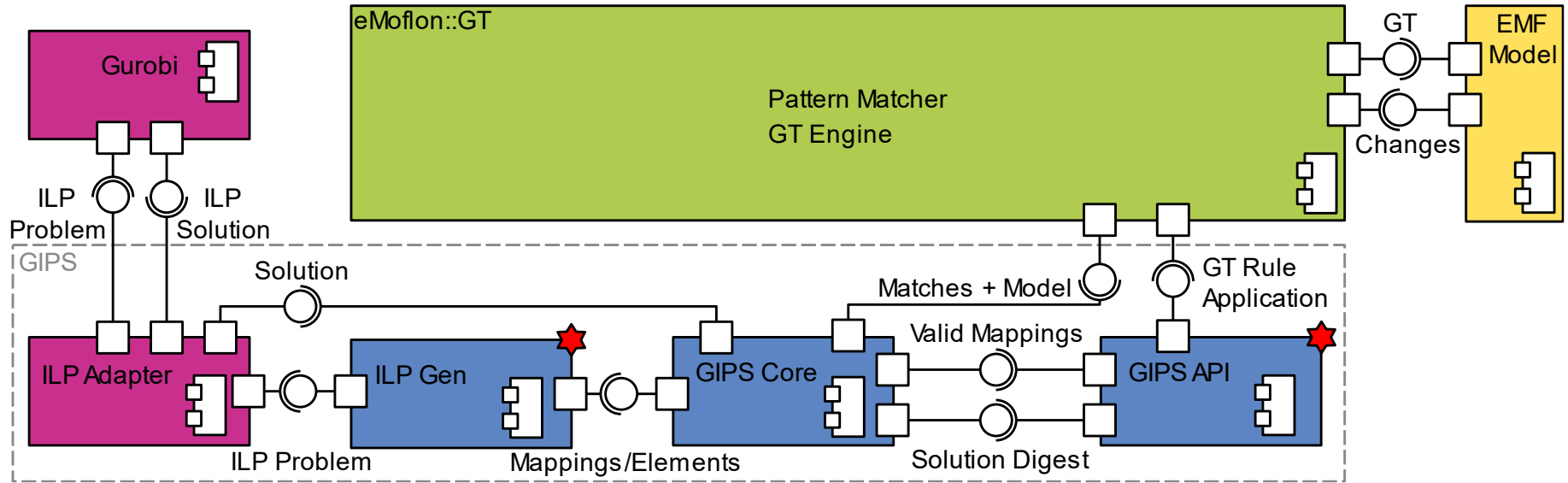
# Our Idea: GIPS (Graph-Based ILP Problem Specification Tool)



★ = generated code



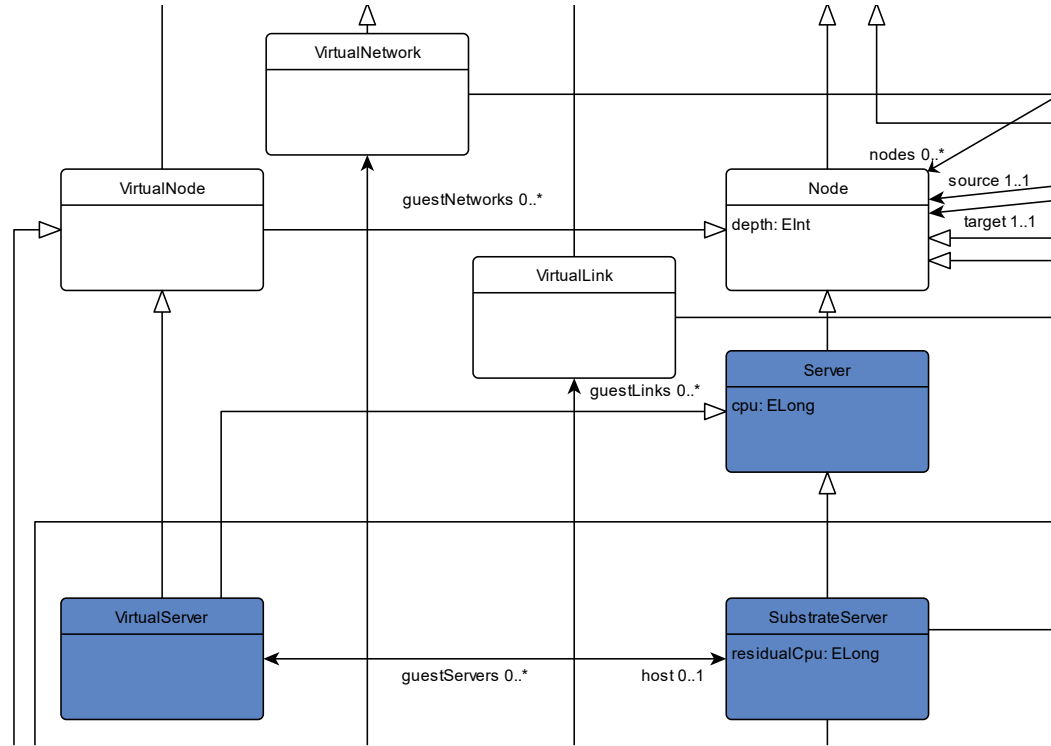
# GIPS: Architecture



★ = generated code



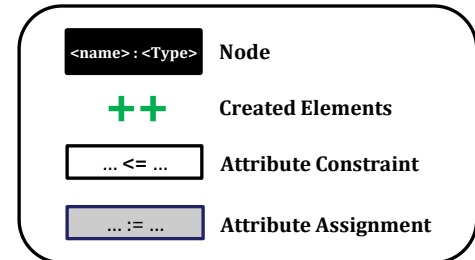
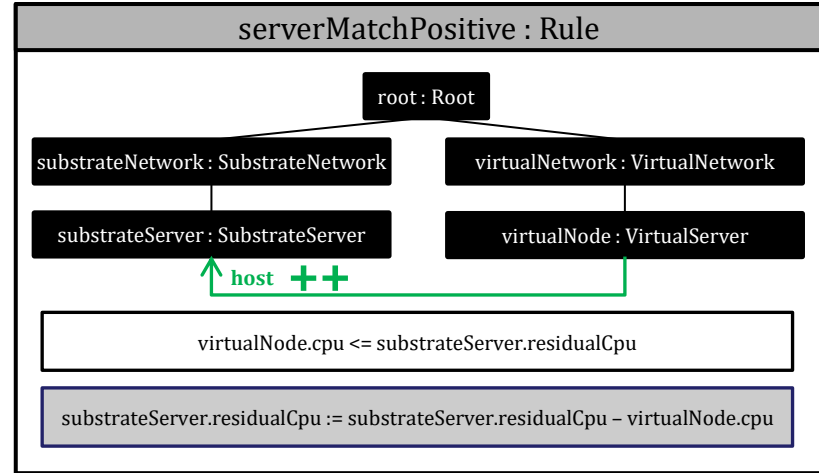
# Example: MdVNE – Metamodel





# GIPSL (GIPS Language) as Example

```
rule serverMatchPositive {
  root: Root {
    -networks -> substrateNetwork
    -networks -> virtualNetwork
  }
  substrateServer: SubstrateServer {
    .residualCpu := substrateServer.residualCpu - virtualNode.cpu
    ++ -guestServers -> virtualNode
  }
  virtualNode: VirtualServer {
    ++ -host -> substrateServer
  }
  substrateNetwork: SubstrateNetwork {
    -nodes -> substrateServer
  }
  virtualNetwork: VirtualNetwork {
    -nodes -> virtualNode
  }
  # virtualNode.cpu <= substrateServer.residualCpu
}
when serverNotMapped
```

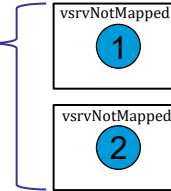


# GIPSL (GIPS Language) as Example

mapping srv2srv with serverMatchPositive;

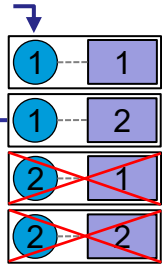
```
constraint -> pattern::vsrvNotMapped {
  mappings.srv2srv
    ->filter(m | m.nodes().virtualNode == self.nodes().virtualServer)
    ->sum(m | m.value()) == 1
}
```

Matches of pattern  
*vsrvNotMapped*



For every match, filter  
*srv2srv* mappings

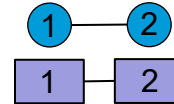
Only one mapping  
must be chosen

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$


```
constraint -> class::SubstrateServer {
  mappings.srv2srv
    ->filter(m | m.nodes().substrateServer == self)
    ->sum(m | m.nodes().virtualNode.cpu)
      <= self.residualCpu
}
```

```
objective srvObj -> mapping::srv2srv {
  (self.nodes().substrateServer.residualCpu / self.nodes().substrateServer.cpu)
    * self.value()
}
```

Example



Virtual Server

Substrate Server

Match



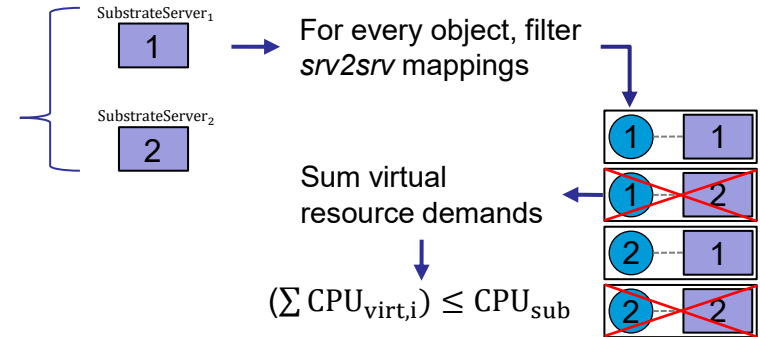
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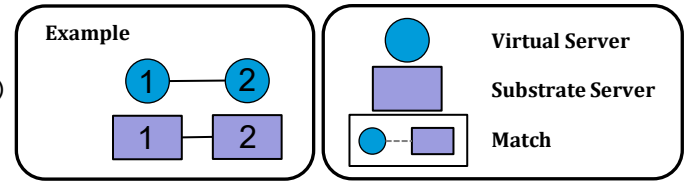
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  mappings.srv2srv
  ->filter(m | m.nodes().substrateServer == self)
  ->sum(m | m.nodes().virtualNode.cpu)
  <= self.residualCpu
}
```

Objects of class *SubstrateServer*



```
objective srvObj -> mapping::srv2srv {
  (self.nodes().substrateServer.residualCpu / self.nodes().substrateServer.cpu)
  * self.value()
}
```



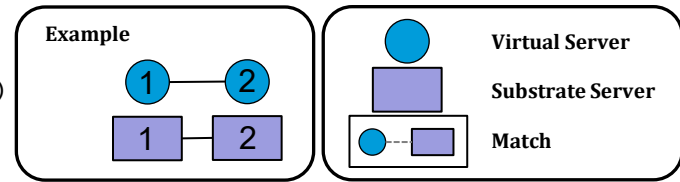
# GIPSL (GIPS Language) as Example

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mapping srv2srv with serverMatchPositive;

constraint -> pattern::vsrvNotMapped {
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constraint -> class::SubstrateServer {
  mappings.srv2srv
    ->filter(m | m.nodes().substrateServer == self)
      ->sum(m | m.nodes().virtualNode.cpu)
        <= self.residualCpu
}

objective srvObj -> mapping::srv2srv {
  (self.nodes().substrateServer.residualCpu / self.nodes().substrateServer.cpu)
  * self.value()
}
```



# Generated ILP Problem (Example)

\ LP format - for model browsing. Use MPS format to capture full model detail.

Minimize

1 srv2srv#3 + 1 srv2srv#2 + 1 srv2srv#1 + 1 srv2srv#0 + \$LINK\_OBJECTIVES

```
objective srvObj -> mapping::srv2srv {  
    (self.nodes().substrateServer.residualCpu / self.nodes().substrateServer.cpu)  
    * self.value()  
}
```

Subject To

\$PATH\_CONSTRAINTS \$SWITCH\_CONSTRAINTS \$NETWORK\_CONSTRAINTS

PatternConstraint30nvsrvNotMapped\_0: srv2srv#2 + srv2srv#1 = 1

PatternConstraint30nvsrvNotMapped\_1: srv2srv#3 + srv2srv#0 = 1

TypeConstraint40nSubstrateServer\_0: 10 srv2srv#3 + 10 srv2srv#1 <= 10

TypeConstraint40nSubstrateServer\_1: 10 srv2srv#2 + 10 srv2srv#0 <= 10

```
constraint -> pattern::vsrvNotMapped {  
    mappings.srv2srv  
    ->filter(m | m.nodes().virtualNode == self.nodes().virtualServer)  
    ->sum(m | m.value()) == 1  
}
```

BOUNDS

Binaries

srv2srv#3 srv2srv#2 srv2srv#1 srv2srv#0

\$SWITCH\_VARS \$LINK\_VARS

```
constraint -> class::SubstrateServer {  
    mappings.srv2srv  
    ->filter(m | m.nodes().substrateServer == self)  
    ->sum(m | m.nodes().virtualNode.cpu)  
    <= self.residualCpu  
}
```

End



# Generated ILP Problem (Example)

\ LP format - for model browsing. Use MPS format to capture full model detail.

Minimize

1 srv2srv#3 + 1 srv2srv#2 + 1 srv2srv#1 + 1 srv2srv#0 + \$LINK\_OBJECTIVES

} Target function

Subject To

\$PATH\_CONSTRAINTS \$SWITCH\_CONSTRAINTS \$NETWORK\_CONSTRAINTS

PatternConstraint30nvsrvNotMapped\_0:  $\text{srv2srv\#2} + \text{srv2srv\#1} = 1$

PatternConstraint30nvsrvNotMapped\_1:  $\text{srv2srv\#3} + \text{srv2srv\#0} = 1$

TypeConstraint40nSubstrateServer\_0:  $10 \text{ srv2srv\#3} + 10 \text{ srv2srv\#1} \leq 10$

TypeConstraint40nSubstrateServer\_1:  $10 \text{ srv2srv\#2} + 10 \text{ srv2srv\#0} \leq 10$

} Map virtual servers once  
} CPU resource constraints

BOUNDS

Binaries

srv2srv#3 srv2srv#2 srv2srv#1 srv2srv#0

\$SWITCH\_VARS \$LINK\_VARS

End

} Mapping variables = binaries



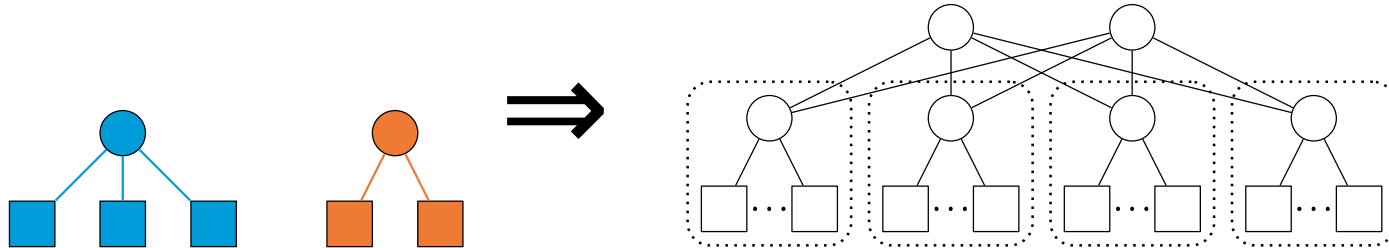
# Evaluation – Research Questions

1. How does our approach compare itself to "hand-crafted" approaches with respect to problem solving **runtime performance**?
2. How much **effort** (for example, *Lines Of Code* (LOC), *Number Of Characters* (NOC)) can really be saved by using our approach to generate an ILP generator compared to implementing one "by hand"?



# Evaluation – Setup

- Task: Virtual Network Embedding (VNE)
- 40 (pseudo-)random generated Virtual Networks (VNs)
  - Embedding takes place one after the other (no batch)
  - No migration
- Data sampled from an industry measurement<sup>[7]</sup>

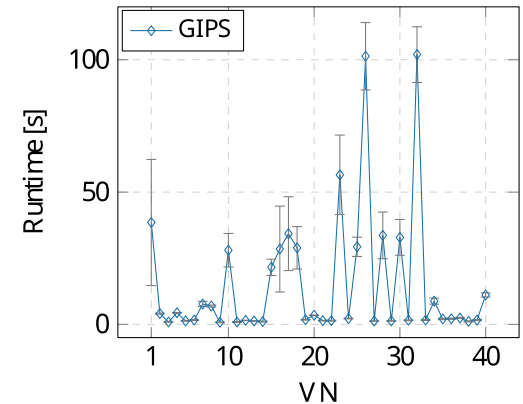
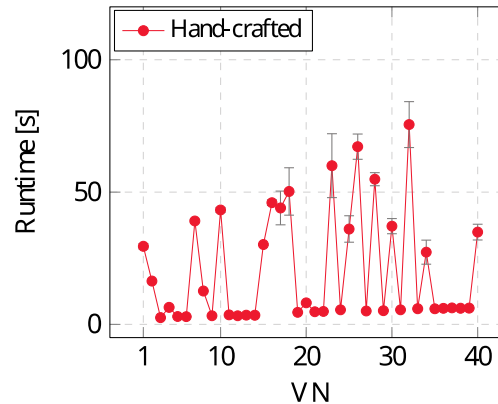




# Evaluation – Results

- Both algorithms were able to embed the 40 VNs successfully
- Quality of the embeddings is approximately equal (w.r.t to the objective)
- Runtime GIPS: ~25% lower

Name	LOC	NOC
Manual	~2000	~91000
GIPS	56+29	3884+1251
Saving	~95%	~94%



# Future Work – GIPSL

- Extension to support non-binary variables (full ILP + MILP + LP)
  - Idea: Real / integer variable support through parametrized rules
- Automated construction of constraints (from given metamodel, GT rules)
  - For example, as hint or auto completion
- Consideration of inter-rule dependencies
- Output of LP files without starting the solver (e.g., for debugging)
- Further evaluation & testing with different scenarios
  - E.g., test scheduling, peer-2-peer overlay networks, transformation tool contest



# References

- [1] Stefan Tomaszek, Roland Speith & Andy Schurr (2021): Virtual network embedding: ensuring correctness and optimality by construction using model transformation and integer linear programming techniques. *Software and Systems Modeling*, pp. 1299–1332, doi:10.1007/s10270-020-00852-z.
- [2] eMoflon::IBeX-GT - <https://emoflon.org/#emoflonIbex>
- [3] eMoflon::Neo - <https://emoflon.org/#emoflonNeo>
- [4] Martin Fleck, Javier Troya & Manuel Wimmer (2015): Marrying search-based optimization and model transformation technology. In: *Proc. of the North American Symposium on Search Based Software Engineering, NasBASE '15*, pp. 1–16.
- [5] Antonio Cicchetti, Davide Di Ruscio, Romina Eramo & Alfonso Pierantonio (2011): JTL: A Bidirectional and Change Propagating Transformation Language. *Software Language Engineering*, pp.183-202, doi:10.1007/978-3-642-19440-5\_11
- [6] HiPE - <https://github.com/HiPE-DevOps/HiPE-Updatesite>
- [7] Siqi Shen, Vincent Van Beek & Alexandru Iosup (2015): Statistical Characterization of Business-Critical Workloads Hosted in Cloud Datacenters. In: *Proc. of the Int. Symposium on Cluster Computing and the Grid, CCGrid '15, ACM*, pp. 465–474, doi:10.1109/CCGrid.2015.60.

